Question	Scheme	Marks	AOs
1.(a)	Mass ratios: $14r$, πr , $14r+\pi r$	B1	1.2
	Distances: $7r$, $\dfrac{2r}{\pi}$, $\bar{x}$	B1	1.2
	Moments equation about *AB*	M1	3.1b
	$14r\times 7r-\pi r\times\dfrac{2r}{\pi}=\left(14r+\pi r\right)\bar{x}$	A1	1.1b
	$\bar{x}=\dfrac{96r}{\left(14+\pi\right)}$	A1	1.1b
		(5)	
(b)	$\tan\theta=\dfrac{\left(\dfrac{\pi r}{14+\pi}\right)}{(14r-\bar{x})}$	M1	3.1b
	θ = 1.25° (3 SF)	A1	1.1b
		(2)	
			(7 marks)

Notes:

1a	B1	Mass ratios correct
	B1	Distances could be measured from a parallel axis
	M1	All terms needed and must be dimensionally correct
	A1	Correct equation (possibly for a parallel axis)
	A1	cao (must be in terms of π and r)
1b	M1	Allow the reciprocal
	A1	cso (from 1.249955….)

Question	Scheme	Marks	AOs
2(a)	$\frac{\mathrm{d}v}{\mathrm{d}t} = v - \frac{1}{10}v^2$	M1	2.5
	$\int \frac{10\mathrm{d}v}{v(10-v)} = \int \mathrm{d}t \quad => \quad \int \frac{1}{v} + \frac{1}{10-v}\mathrm{d}v = t$	M1	2.1
	$\ln v - \ln(10-v) = t + (C)$	A1	1.1b
	Use of initial conditions or appropriate limits, and solve for *v*	M1	2.1
	$v = \frac{20\mathrm{e}^t}{2\mathrm{e}^t + 3}$ *	A1*	1.1b
		(5)	
2(b)	$\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{20\mathrm{e}^t}{2\mathrm{e}^t+3} \Rightarrow x = \int \frac{20\mathrm{e}^t}{2\mathrm{e}^t+3}\mathrm{d}t$ **OR** $v\frac{\mathrm{d}v}{\mathrm{d}x} = v - \frac{1}{10}v^2 \Rightarrow x = \int \frac{10}{(10-v)}\mathrm{d}v$	M1	2.5
	$x = 10\ln(2\mathrm{e}^t + 3) \quad (+C)$ **OR** $x = -10\ln(10-v) \quad (+D)$	A1	1.1b
	Use of initial conditions or appropriate limits to find *x*	M1	2.1
	$10\ln\left(\frac{3+2\mathrm{e}}{5}\right)$ *	A1*	1.1b
		(4)	
		(9 marks)	

Notes:

2a	M1	Choosing appropriate derivative form for *a*
	M1	Separate variables and split into partial fractions
	A1	Correct equation (*C* not needed)
	M1	Using initial conditions to find a value for *C* and inserting into expression or substituting in limits if using a definite integral and solve for *v*
	A1*	Correct derivation of **given answer**
2b	M1	Attempt to integrate wrt *t* or *v*
	A1	Correct equation (*C* or *D* not needed)
	M1	Using initial conditions to find a value for *C* or *D* and inserting into expression or substituting in limits if using a definite integral
	A1*	Correct derivation of **given answer**

Question	Scheme	Marks	AOs
3(a)	Use of a semicircular element	M1	2.1
	$\delta A \simeq \pi x \delta x$	A1	1.1b
	$\delta m \simeq \pi x \delta x \times kx$ $(= \pi k x^2 \delta x)$	M1	2.1
	$M = \int_0^a \pi k x^2 \mathrm{d}x$	M1	2.1
	$k = \dfrac{3M}{\pi a^3}$ *	A1*	1.1b
		(5)	
3(b)	Use of $\bar{x} = \dfrac{1}{M}\int x \, \mathrm{d}m$	M1	3.4
	$= \dfrac{1}{M}\int_0^a \dfrac{2x}{\pi} \pi k x^2 \mathrm{d}x$	A1	1.1b
	Substitute for M or k and integrate	M1	3.4
	$\bar{x} = \dfrac{3a}{2\pi}$	A1	1.1b
		(4)	
			(9 marks)

Notes:

3a	M1	Use of appropriate element (may be implied)
	A1	Correct expression for area of element (may be implied)
	M1	Use of proportionality
	M1	Integrating with correct limits
	A1*	GIVEN ANSWER
3b	M1	Use the model with correct general formula
	A1	Correct integral
	M1	Use the model to complete the equation
	A1	cao

Question	Scheme	Marks	AOs

4(a)	Resolving vertically	M1	2.1
	$T\cos\theta = mg$	A1	1.1b
	Equation of motion horizontally	M1	2.1
	$T\sin\theta = m(a+x)\omega^2\sin\theta$	A1	1.1b
		A1	1.1b
	Use of Hooke's Law	M1	1.1b
	$T = \dfrac{kmgx}{a}$	A1	1.1b
	Overall strategy to solve problem by eliminating T	DM1	3.1a
	and x and solving for $\cos\theta$	DM1	3.1a
	$\cos\theta = \dfrac{(kg - a\omega^2)}{ka\omega^2}$ *	A1*	2.2a
		(10)	
4(b)	$\theta < 90^\circ \Rightarrow \cos\theta > 0 \Rightarrow \cos\theta = \dfrac{(kg - a\omega^2)}{ka\omega^2} > 0$	M1	2.1
	$\omega < \sqrt{\dfrac{kg}{a}}$ *	A1*	1.1b
		(2)	
		(12 marks)	

Notes:

4a	M1	Correct no. of terms with T resolved
	A1	Correct equation
	M1	Correct no. of terms with T resolved and correct acceleration component
	A1	Correct equation with at most one error
	A1	Correct equation
	M1	Use of Hooke's Law
	A1	Correct expression
	DM1	Dependent on previous 3 M marks
	DM1	Dependent on previous M mark
	A1*	GIVEN ANSWER
4b	M1	Clear argument
	A1*	GIVEN ANSWER

Question	Scheme	Marks	AOs
5(a)	Equation of motion along string when string goes slack	M1	3.1b
	$mg\cos\alpha = \dfrac{mv^2}{a}$	A1	1.1b
	Conservation of energy	M1	3.1b
	$\dfrac{1}{2}m\left(\dfrac{22ag}{5} - v^2\right) = mga(1+\cos\alpha)$	A1 A1	1.1b 1.1b
	Overall strategy to use the equations to eliminate $\cos\alpha$ and solve for v	M1	3.1b
	$v = \sqrt{\dfrac{4ag}{5}}$ *	A1*	1.1b
		(7)	
5(b)	Only force acting is *mg* downwards oe (e.g. $mg = ma$)	B1	2.4
	g, vertically downwards	B1	1.1b
		(2)	
5(c)	Conservation of energy	M1	3.1b
	$\dfrac{1}{2}m\left(w^2 - \dfrac{4ag}{5}\right) = mga \times \dfrac{4}{5}$	A1	1.1b
	$w = \sqrt{\dfrac{12ag}{5}}$	A1	1.1b
	ALTERNATIVE		
	Conservation of energy	M1	3.1b
	$\dfrac{1}{2}m\left(\dfrac{22ag}{5} - w^2\right) = mga$	A1	1.1b
	$w = \sqrt{\dfrac{12ag}{5}}$	A1	1.1b
		(3)	
5(d)	There would be some air resistance on the stone	B1	3.5b
		(1)	
			(13 marks)

Notes:

5a	M1	Correct no. of terms with *mg* resolved and correct acceleration component
	A1	Correct equation

	M1	All terms needed and dimensionally correct
	A1	Correct equation with at most one error
	A1	Correct equation
	M1	Eliminate $\cos\alpha$
	A1*	GIVEN ANSWER
5b	M1	Clear explanation
	B1	cao
5c	M1	All terms needed and dimensionally correct
	A1	Correct equation
	A1	cao
5d	B1	Penalise extra wrong answers

Question	Scheme	Marks	AOs
6(a)(i)	$T_1 = mg + T_2$	M1	2.1
	Use of Hooke's Law to give equation in one unknown	M1	3.1a
	$\frac{\lambda e}{l} = mg + \frac{3\lambda(2l-e)}{l}$	A1	1.1b
	$AO = l + e = l + \frac{l}{4\lambda}(mg + 6\lambda) = \frac{l(10\lambda + mg)}{4\lambda}$ *	A1*	1.1b
		(4)	
(ii)	Use of $AO < 3l$, $\frac{l(10\lambda + mg)}{4\lambda} < 3l$	M1	3.1a

	$\frac{1}{2}mg < \lambda$ so least possible value of $k = \frac{1}{2}$	A1	1.1b
		(2)	
6(b)	$\lambda = 2mg \Rightarrow AO = \frac{21l}{8}$	B1	1.1b
	Equation of motion for P when a distance x from O, towards A $T_1 - mg - T_2 = m\ddot{x}$	M1	3.1a
	$\frac{2mg}{l}\left(\frac{13l}{8} - x\right) - \frac{6mg}{l}\left(\frac{3l}{8} + x\right) - mg = m\ddot{x}$	A1	1.1b
		A1	1.1b
	$-\frac{8g}{l}x = \ddot{x}$ so SHM with $\omega^2 = \frac{8g}{l}$	M1	3.1a
	Period $= \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{l}{8g}}$ *	A1*	2.2a
		(6)	
6(c)	$U = a\sqrt{\frac{8g}{l}} \Rightarrow a = U\sqrt{\frac{l}{8g}}$	B1	1.1b
	Use of $a \leqslant \frac{3l}{8} \Rightarrow U\sqrt{\frac{l}{8g}} \leqslant \frac{3l}{8}$	M1	3.1a
	$U = 3\sqrt{\frac{gl}{8}}$	A1	1.1b
		(3)	
			(15 marks)

Notes:

6a(i)	M1	All 3 terms needed
	M1	Use of Hooke's Law to solve the problem
	A1	Correct equation
	A1*	GIVEN ANSWER
(ii)	M1	Using fact that bottom string is stretched
	A1	cao
6b	B1	cao
	M1	All terms needed
	A1	Correct equation with at most one error
	A1	Correct unsimplified equation
	A1	Correct simplified equation with conclusion
	A1*	GIVEN ANSWER

6c	B1	cao
	M1	Correct method
	A1	cao

Question	Scheme	Marks	AOs
7(a)	Mass ratios: $\frac{2\pi a^3 k\rho}{3}, 2\pi a^3\rho, \frac{\pi a^3\rho}{3}$ (hemisphere,cylinder,cone)	B1	1.2
	Distances from base : $\frac{5a}{8}, 2a, \frac{13a}{4}$ (hemisphere,cylinder,cone)	B1	1.2
	Moments equation (about base)	M1	3.1a
	$\left(\frac{2\pi a^3 k\rho}{3}\times\frac{5a}{8}\right)+\left(2\pi a^3\rho\times 2a\right)+\left(\frac{\pi a^3\rho}{3}\times\frac{13a}{4}\right)$ $=\left(\frac{2\pi a^3 k\rho}{3}+2\pi a^3\rho+\frac{\pi a^3\rho}{3}\right)\bar{x}$	A1	1.1b
	$\bar{x}=\frac{a(5k+61)}{(8k+28)}$ oe	A1	1.1b
	Use of appropriate inequality, $\bar{x}<a$ to solve the problem	M1	3.1a
	$k>11$*	A1*	1.1b
		(7)	
7(b)	When $k=11$, $\bar{x}=a$ i.e. c of m is at centre of hemisphere	B1	3.2a
	Weight acts through pt. of contact with plane	B1	2.2a
	Moments about pt. of contact => does not move oe	B1	2.2a
		(3)	
			(10 marks)

Notes:

7a	B1	Correct mass ratios
	B1	Correct distances
	M1	All four terms, dimensionally correct. May use a parallel axis
	A1	Correct unsimplified equation with at most one error
	A1	Correct unsimplified expression for $\bar{x}$
	M1	Correct for their $\bar{x}$
	A1*	GIVEN ANSWER correctly obtained
7b	B1	Clear explanation

	B1	Clear explanation
	B1	Clear explanation and conclusion